THE SCHOOL SCALE MODEL

LEARNING AND EVALUATION SITUATION

Authors: Michael Seccareccia, Andrew Iacobo, Stephen McCarthy

OVERVIEW

COURSE AND TIME FRAME			
Course:	Mathematics	Cycle and year:	Cycle 1 Year 2 (Gr 8)

PEDAGOGICAL INTENTION – CONTEXT – BRIEF DESCRIPTION

The complex task for the "School Scale Model" LES starts with a simple premise: the administration has asked your class to create scale models of the school to display in the lobby. Your students will work in groups to perform the task of building these scale models. The complexity in this task as much as possible will arise from natural constraints of the situation.

The Learning and Evaluation Situation built around the above complex task centres around the many proportional situations (rates, ratios, scale factors, etc.) that are part of the Grade 8 curriculum. The LES shows students how proportional situations can be applied in a real-life context through an engaging project. It is our pedagogical intention to have students connect their learning of mathematical concepts implicitly through project work and explicitly through a written reflection.

The LES also brings in mathematical issues that arise in real-world problems: budgeting, allocating resources, and finding unknown measurements. Some of these are also addressed throughout the learning activities.

Detailed descriptions of the complex task and learning activities are attached.

COMPLEX TASK – PROCEDURE

To complete their scale model of the school, the students must work in groups to perform the following steps:

- Measure and record the actual dimensions of the school;
- Choose a ratio of similarity and draw a scale drawing as a base for their model;
- Choose from available materials, working with a set budget and material costs;

- Build the scale models; and
- Write a task report to reflect on and voice their learning.

Subject Specific Competencies Targeted

Mathematics Competency 1: Solves a situational problem

The complex task fits the MELS definition of a situational problem, as students must solve a mathematical problem (constructing a scale model) in a real-life situation given imperfect information. This is not something the students are likely to have done before.

Mathematics Competency 2: Uses mathematical reasoning

The learning activities will develop mathematical reasoning around the area of proportional situations by offering a variety of concepts for students to explore and reasoning-based activities and problems for students to perform and solve.

Mathematics Competency 3: Communicates by using mathematical language

Students will develop their mathematical communication by working in groups, as the need to find a common understanding of the problem will spur the development of mathematical vocabulary, especially related to proportional situations. As well, the task report demands that students communicate in mathematics how they did the complex task.

Cross-Curricular Competencies Targeted

Cross-curricular competency 1: Uses information

Students will need to gather information for the complex task from sources around them, for example by taking measurements of their school. They may need to create a systematic process for doing so while working with a group. They will need to put this information to use in constructing a model.

Cross-curricular competency 2: Solves problems

Students will need to analyze the components of their situation to determine how to construct the model (e.g. what scale? What materials?), test possible solutions (multiple drafts, tests of construction materials), and keep their approach flexible to adapt to their learned experiences as they solve the problem. In the learning activities, students will develop their problem-solving skills relating to proportional situations.

Cross-curricular competency 5: Adopts effective work methods

Similarly to cross-curricular competency 2 above, students will need to consider all aspects of the complex task in evaluating how to carry it out. They may need to adjust their approach to account for issues that arise. Effective work methods will be especially important as students learn to work in groups. Teaching in the learning activities will be targeted at helping students improve their independence and effective work habits.

Broad Area of Learning Targeted

Broad Area of Learning: Personal and Career Planning

The LES touches on many aspects of Personal and Career Planning and will help students develop their learning in this area. The BAL has three focuses of development:

- *Adoption of strategies related to a plan or project.* The task is set up so student groups are given the constraints of the situation and must determine a strategy to proceed. Similarly to cross-curricular competency 5, the group nature of the task makes the adoption of effective strategies important.
- *Self-knowledge and awareness of his/her potential and how to fulfill it.* By integrating many different types of tasks into the complex task (i.e. finding measurements, drawing, calculating, thinking abstractly in mathematics, and hands-on construction), the complex task can help students learn about their strengths and weaknesses in these things.
- *Familiarity with the world of work, social roles, and occupations and trades.* Several elements in the complex task mirror real-world occupations (i.e. surveyor, accountant, contractor) at a Secondary 2 level. Perhaps the most applicable to students is the budgeting component, which is important to many occupations.

Situation in Progression of Learning

The LES draws mainly upon the "Understanding and analyzing proportional situations" portion of the QEP Progression of Learning in Secondary Mathematics. It aims to cover all the material appropriate for the Secondary Cycle 1 Year 2 level in the chart on the next page.

Understanding and analyzing proportional situations

- → Student constructs knowledge with teacher guidance.
- ★ Student applies knowledge by the end of the school year.
- Student reinvests knowledge.

	Elementary	Secondary				
		Cycle One		Cycle Two		
	6	1	2	3	4	5
1. Calculates						
a. a certain percentage of a number	→	★				
b. the value corresponding to 100 per cent		→	★			
2. Recognizes ratios and rates		→	★			
3. Interprets ratios and rates		→	★			
4. Describes the effect of changing a term in a ratio or rate		→	★			
5. Compares						
a. ratios and rates qualitatively (equivalent rates and ratios, unit rate)		→	★			
b. ratios and rates quantitatively (equivalent rates and ratios, unit rate)		→	★			
6. Translates a situation using a ratio or rate **Note** : Situations involving ratios and rates are enriched in Secondary Cycle Two (similarity ratio, metric relations, etc.).		→	★			
7. Recognizes a proportional situation using the context, a table of values or a graph		→	★			
8. Represents or interprets a proportional situation using a graph, a table of values or a proportion		→	★			
9. Solves proportional situations (direct or inverse variation) by using different strategies (e.g. unit-rate method, factor of change, proportionality ratio, additive procedure, constant product [inverse variation])		→	★			
10. Establishes relationships between first-degree or rational functions and proportional situations (direct or inverse variation)				★		

STRUCTURE OF THE LES

❶ *PREPARATION – INTRODUCING THE COMPLEX TASK*

The complex task of building a scale model is not something that students should be expected to do in one day, and indeed we recommend four class periods be dedicated to introducing, scaffolding, providing group work time and presenting finished products. This process can be started before the Learning Activities or part-way through the set of learning activities at the teacher's discretion.

The complex task's context (an administration request for scale models of the school) can be introduced very simply and can be used to motivate the LES. The class and teacher can then have a discussion around students initial understandings of how to complete this project, and students can even take their initial measurements of the school (both to get excited and to serve as a reference point for future work on the complex task).

❷ *LEARNING AND PERFORMANCE – THE LEARNING ACTIVITIES*

The four learning activies are summarized here in our recommended order. We also recommend taking a class out between the activities from time to time to continue the complex task.

Learning Activity 1: Ratios

The purpose of this LA is to take a hands-on approach to learning by exploring the topic of ratios. In this learning activity, students will examine ratios of lengths on the human face, with the goal of linking mathematics to the real life situation of the human body.

Learning Activity 2: Rates

The purpose of this lesson is to facilitate students' understanding of the concept of rates and how they relate to ratios, and to do various computations with rates and draw on real-life application of rates. Students will apply this to budgeting and the cost of living, for example by calculating the costs of items bought and other living expenses.

Learning Activity 3: Proportions

The purpose of this LA is to have students construct their own meanings around proportional situations. Starting with the theories of Vitruvius on ratios in the Vitruvian Man, students will create and test hypotheses by measuring lengths on their own bodies.

Learning Activity 4: Scale Factors

This is a quick LA to expose students to the concept of scale factors, which are central to the complex task. Students will observe and measure objects in their classroom and draw them according to scale factors.

❸ *INTEGRATION & REFLECTION – COMPLETING THE COMPLEX TASK*

Once students have completed all the learning activities and are ready to build their models, the complex task is almost complete and the students will be busy integrating their learning by completing the task. This process includes a reflective portion where students must write a task report that includes a description of their solution procedure, how they worked as a team, a mathematical explanation of their work, and reflection questions. (See the detailed plan for our suggestions in this area.)

We highly recommend a follow-up discussion, perhaps arising out of the task report questions, in which students discuss what they learned from the project, not only about the mathematics of proportional situations but also about what they could take away from this project to their own lives. Sample prompts for such a discussion include:

- What issues arose while planning and building the models, and how did groups deal with them?
- How could you take this knowledge to another subject or to your future life?

LEARNING ACTIVITY 1: RATIOS

Duration: 2 – 3 classes of 60 minutes each.

OVERVIEW AND BACKGROUND:

The purpose of this lesson is to enable students to have a hands-on approach to learning by exploring the topic of ratios. In this learning activity, students will examine ratios on the human face. In doing so, students will end up discovering that many ratios are approximately equal, leading to what is often called the Golden Ratio. The purpose of teaching the golden ratio is to link mathematics to real life situations, such as the human body, architectural design and other proportional structures found in nature. By the use of manipulatives such as metric rulers, students will be experimenting with objects found in their immediate environment. Before having students perform this learning activity, they will be given a lecture on ratios and need to be familiar with the different mathematical lexicons. Ratios were extremely important in the Renaissance and other historical eras, such as in paintings and sculptures of Leonardo da Vinci and his contemporaries. This lesson also entails a cross-curricular component to it, in that it connects mathematics to students' past and present environment.

PURPOSE:

In this lesson, students will learn what a ratio is and use ratios to describe proportional situations (i.e. various word problems relating to measurements/relations and finally learning-based activity: head to body ratio, entitled the golden ratio).

This lesson focuses on Mathematics Competency 2: *Uses Mathematical reasoning through different concepts and processes.*

MATERIALS:

- Laptop (PowerPoint slideshow)
- Notebooks
- Handouts (to be distributed)----important for drill and practice routine
- Workbook handouts (Panorama)
- Students will be encouraged to use calculators to do various computations (hands on approach).
- SmartBoard
- Reference sheet will be utilized so that students can use it as a reference point in conjunction with the lesson being taught
- Ruler
- Images, pictures of statues or art forms of people (so that they can experiment with the golden ratio)

LESSON OUTLINE:

1st class:

- While taking attendance, students will work on “the problem of the day”. It will be an open-ended question on ratios as an introduction to the golden ratio to complement what the week's lesson will entail.
- This week’s lesson will begin by introducing key vocabulary words which students are expected to know by the end of the lesson (i.e. ratios, proportions, relationship, percentage, decimal, distance measurement, Fibonacci sequence and other (if time permits).
- The lesson will begin with a review of key terms and how to apply them to word problems and a question period will follow.
- The above lesson is preliminary in scope, in order to initiate my students into becoming familiar with ratios in more depth and before exploring the golden ratio activity (which is to come next).
- Work through various word problems in workbook respectively. Have students work in teams to dissect various word problems
- Ask for class’ input/question/comments and work through the problems collaboratively. Students will work on the handouts that I will then distribute in groups.
- Assign as homework to explore the content further. Students will complete the handout in workbook for homework and be asked to bring a picture/image of someone (i.e. historical figure, celebrity...) and ruler so they can work on the golden ratio activity next class.

2nd class:

- Check for understanding: The key question to be addressed will be: How are ratios used in the real world?
- Review old mathematical notions and introduce new ones (i.e. ratios, proportions, relationship, percentage, decimal, distance measurement, Fibonacci sequence) from last class.
- We will correct last classes' homework together in the workbook/stencils.
- Once the 1st component of the lesson is completed (teaching and independent work), students will work on the activity component of the lesson. Students were asked to bring in a picture and ruler to class. To begin, the class will be divided into groups of five (arrange desks in square form if need may be) (5 students per group, 1 group of 6).
- Instructions will be given on activity to be done before “stations log” (with tables) is distributed to each student
- There will be 5 working stations (instructions provided on handout)
- Each student is going to measure the appropriate measurement found on the instructions (i.e. chart).

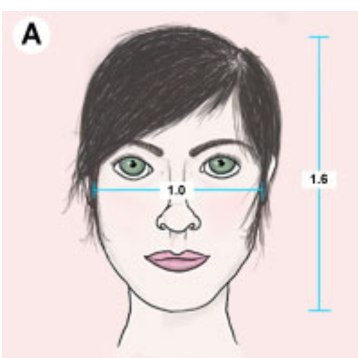
- The handout from one of the stations can be finished for homework (if students did not have enough time to complete it). They will be picked up at the beginning of the next class.

EXPLORATION/MOTIVATION:

- Getting input/feedback from students (asking questions and working collaboratively to dissect new problems)
- To activate prior knowledge and help students review previously learned vocabulary/problems/applications
- Making associations to real life situations, making students appreciate concepts of ratios and the important part that mathematics plays in their lives.
- Students will be encouraged to use a reference page (formula sheet) in the workbook so as to allow them to make associations with the material being taught. This approach will facilitate problem solving since they will have all the tools required at their disposal. First and foremost, the objective of my lesson is to show my students that this topic can be made fun and does not have to only entail definitions and formulas but also involves relationships and reasoning that connect to real-life situations.

EVALUATION:

The evaluation of the lesson will be carried out through observation and how each individual student participates with the activity presented to him or her. My duty as a teacher will be to motivate each student and make sure everyone has a role to play. My learning activity will test students' knowledge on previously seen material. Ultimately, the purpose of this activity is to allow students to appreciate what mathematics entails and create interest in what is being taught. Before getting into the activity component of the lesson, the beginning of the lesson will be approached in a traditional manner. Nevertheless I will emphasize the importance of drill and practice routine in mathematics; the more students practice the concepts, the easier it will become for them. My aim is to modify the lesson throughout the week depending on how students learn. For instance, I may use visual cues to make the concepts easier to grasp (videos, manipulatives and getting students actively involved by writing their answers on the board).



I will circulate around the class, paying particular attention to the discussions as well as individual efforts being conducted by each group to the task at hand (while working on problems). My duty as a teacher will be to spark interest on the part of each student and make sure everyone actively participates in the lesson given.

METHOD OF INSTRUCTION/MOTIVATION

- Lecture given (practice concepts on board at beginning of class or correct homework at beginning of class)
- Class collaboration (work in workbook or/and worksheets); getting input/feedback from students (asking questions and working collaboratively to dissect new problems)
- To activate prior knowledge and help students review previously learned vocabulary/problems/applications.

REFERENCES:

http://www.geom.umn.edu/~demo5337/s97b/

Learning Activity 2: Rates

Duration: 2 - 3 classes of 60 minutes each.

Overview and Background:

A rate is a special type of ratio whereby we use words such as per, at or each. We use rates in our everyday life such as when comparing the number of DVDs to the cost of DVDs. The latter is an example of a comparison of number measurements to money measurements. Another similar example is when comparing miles to gallons since we are using different unit measures. Students will learn to apply these concepts to numerous real world mathematical applications.

Purpose:

In this lesson, students will understand the concept of rates and how they relate to ratios, and will do various computations with rates while drawing on real-life application (practical application exercise). The practical application component of the lesson will demonstrate knowledge of planning for living costs by calculating the costs of items bought and other living expenses (i.e. individual's budget plan).

This lesson focuses on Mathematics Competency 2: Uses Mathematical reasoning through different concepts and processes.

Materials:

- Laptop (PowerPoint slideshow)
- Notebooks
- Handouts (to be distributed)----important for drill and practice routine
- Workbook handouts (Panorama)
- Students will be encouraged to use calculators to do various computations (hands on approach).
- SmartBoard
- Reference sheet will be utilized so that students can use it as a reference point in conjunction with the lesson being taught

Lesson outline:

1st class:

- I will start off by writing examples of rates on the SmartBoard. Have students form a group and come up with two rates to share with the class. They will present their findings with the class (their different examples of rates) and record all other examples from other students in their notebook.

- Students will take notes for the first part of the lesson. I will begin by introducing key vocabulary words which students are expected to know by the end of the lesson. This lesson is an introduction to rates, before getting into the activity part 2 of the lesson.
- Students will be assigned word problems on the stencil provided. Have students work in teams to dissect the various word problems. Students will continue to practice solving word problems/real-life applications involving rates and unit rates for the remainder of the class.
- We will discuss the different between rate and unit rate (i.e. a rate is a comparison of two quantities using different types of measures such as kms to gallons, pounds per cm whereas, a unit rate is when a measure of quantity is compared to a single unit of another.)
- We will go over the examples together as a class and have students volunteer to give their answers. Homework will be assigned depending on how much material we covered so far (homework assignment subject to change).
- For next class, students will bring in flyers with commercial products and determine the best bargain price of specific items, which will be explained next class.

2nd class:

- Check for understanding: Key questions to be addressed will be:
    - Give some examples of rates.
    - Name a situation when computing a rate might be helpful.
    - Why must you be consistent when setting up a proportion (fraction)?
- Students will use the flyers they brought to class to work on the assignment. I will give instructions prior to having them begin the assignment. Basically, they will determine the best bargain price of specific items, which they will eventually present to the class.
- This will draw on students buying habits and why price is an important factor and how to determine which product is the best buy. Some questions to think about here are: What do good shoppers do to save money (possible answers: look for sales, coupons).
- Students will glue the items from the flyers into separate columns whereby they will put the cheapest item together and the priciest items in a different column. They will have to calculate how much one store charges as compared to the other, how many savings they will have and other monthly incomes.
- The purpose of this part of the lesson is to make students responsible in their own learning. Since students will one day need to know how to save money and budget their costs, this activity is an application of math concepts similar to what they will experience in the outside world. They will figure out their gross and net income and break down their income into monthly budgets. While putting together this activity, students will use a variety of digital tools, including graphs/charts/tables, Excel spreadsheet (if need be), and presentation posters to display their calculations and data, which they will present to the class the next day.

3rd class:

- Students will present their posters.

EVALUATION:

The evaluation of the lesson will be carried out through observation and how each individual student participates with the activity presented to him or her. The beginning of the lesson will revolve around a series of problems that build on one another and I plan to ask many questions to assess if students have properly understood the material and if progress has been made. For the second component of the lesson (budgeting activity), students will be evaluated on how well they collect and analyze data to come up with informed decisions (coming up with the best possible budgeting monthly costs/savings solving authentic problems using digital tools (i.e. ICT).

METHOD OF INSTRUCTION/MOTIVATION

- Lecture given (practice concepts on board at beginning of class or correct hw at beginning of class)
- Class collaboration (work in workbook, worksheets and group activity); getting input/feedback from students (asking questions and working collaboratively to dissect new problems)
- To activate prior knowledge and help students review previously learned vocabulary/problems/applications

Learning Activity 3: Proportions

Duration: 4 classes of 60 minutes each.

Overview and Purpose:

Having been introduced to ratios and rates, the students will now venture into the realm of proportions. They will use their knowledge acquired throughout the chapter, and build on said knowledge through various activities and lessons. Since proportions are an extension of ratios, so too will the activity be. By examining Leonardo da Vinci's drawing of the Vitruvian Man, students will get a first taste of proportions, comparing the ratios of their own limbs to those of the theories surrounding the men and women who lived roughly 2000 years ago! The activity aims to get the students directly involved with the hundreds and thousands of proportions that they unknowingly come across every day.

The students will have access to various learning material to supplement their learning process. Such materials include teacher-made videos, handouts, workbook pages, and other online material.

This LA focuses on Mathematics Competency 2: Uses mathematical reasoning.

Materials:

- Laptop + Internet
- Drawing Tablet + Screen recording software
- Workbook
- Stencils
- String + Tape measure
- Ruler
- Calculators

Lesson Outline:

1st Class:

- The students will begin the class with a "problem of the day". This question will test their retention of info and understanding related to ratios and rates. It will also provide time to take attendance, and allow for minor yet important evaluation. The teacher can come up with this on the spot in order to remain flexible.
- Following the bell work, the students will be instructed to pair up, in order to complete the Vitruvian Man activity. The activity itself combines borrows aspects from the science curriculum in that it is an experiment, and must be treated as such. The students will of course have guidance from the teacher, who will provide insight into the scientific method, and a

brief explanation of proportions. Since this is an experimental activity, it is meant to provoke definitions and thoughts from the students before the teacher explains the topic thoroughly.

- Though the tasks may seems trivial, the activity really allows the students to grasp the idea that proportions, though they may be comparing seemingly unrelated substances, may provide comparative insight. It also allows the students to appreciate the types of ratios that can and cannot be compared.
- Materials necessary will include string, and a ruler (or tape measure, for more precision), as well as a paper to take down their findings and draw conclusions.
- The activity will come to an end with a class discussion revolved around the general findings of the activity, and the abundance of proportions that surround us in day-to-day life.
- For homework, the students will be responsible for watching and taking notes on the "Proportions" math video, made by the teacher. The video is available here: http://www.youtube.com/watch?v=6eX0cQzhcQ8

2nd Class:

- The second class on proportions will start with another problem of the day, once again left to the teacher. This question should cover material that was presented in the video, and can be specific to proportions, or touch on rates and ratios as well.
- Having established a foundation of knowledge through the Vitruvian activity and the video, the main focus of the class will be Directly Proportional Situations.
- The students will be given the handout (see attached) and will be led through the first part by the teacher.

3rd Class:

- The 3rd class on proportions will cover inverse proportionality. Building off of their practice with directly proportional situations, the teacher can introduce the concept of inverse proportionality. The class can, as per usual, start with a question of the day covering the previous day's material.
- A typical introductory example of inverse proportionality involves explaining what would happen if a lottery winner were to split the winnings with one, two, three, etc. people. Alternatively, dividing a pizza amongst friends works as well as an elementary example that relays the concept.
- Students will then be given a stencil, such as the one given in the last class, to follow along as the teacher discusses the topic, while calling on students for insight.
- Should time permit, the lesson on inverse proportionality could be followed up by a short quiz on proportionality. The quiz is meant to test the students' broad understanding of the topic, and can also be used to guide them (it need not be taken as a serious quiz, but more of a practice, guided by the teacher). A sample of such a quiz is included in the attachments.

4th Class:

- The fourth class on proportions is to be used, if necessary, as a review period. The students can be paired up, work in groups, or work individually, and complete the review package, which covers ratios, rates, and proportions. It is meant to give the students an indicator of where their level of understanding currently is. Below is an example of such a review package:

Learning Activity: Scale Factor and Similar Figures

Duration: 1 class of 60 minutes.

Overview:

Understanding scale factor is essential in order to compare similar figures, and conduct similarity transformations. Relative to this LES, knowledge of scale factors will allow the students to complete the Complex Task, which involves building a scale model of the school. Similarity transformations are therefore left out for the time being. In a sense, then, this introductory class on similar figures is really an intermediary class – its purpose is to enlighten the students so that they may complete the task, but it is certainly not as comprehensive as can be. Further lessons on the topic will take place after the completion of the CT.

Class may begin with a question of the day, keeping with the established routine. The students will have been asked to watch the video on similar figures, available here: http://www.youtube.com/watch?v=zzupQp4Z7aA. While the students complete the question of the day, the teacher can walk around in order to verify whether or not the notes were taken.

The students will be given the worksheet attached, and will follow along as the teacher leads them through the definitions and examples of similar figures. The stencil is aimed at providing a rudimentary understanding, using basic shapes and concepts.

Once the worksheet is completed (~30 minutes), the students will be separated into groups (like they will be for the Complex Task). Each group will be assigned a number (representing a scale factor), and asked to find 2 objects in the class that they can draw according to their scale factor, while indicating measurements and calculations. This is a preparatory activity for the CT. The teacher can collect the drawings in order to evaluate whether or not the students are ready for the CT.

Complex task: A school scale model

Introduction

The administration has asked your class to create scale models of the school to display in the lobby. Students will work in groups to perform the complex task of building these scale models.

This complex task uses students' knowledge of rates, proportions, and scale factors in several ways: to measure and make a scale drawing of the school, to calculate areas on their model in order to determine how many materials they will need, and to properly budget their model. It will also touch on the following Grade 8-specific math skills: finds unknown area measurements, recognizes solids that can be split into basic solids. Finally, it will use budgeting skills including many math skills the students have built up in elementary school and Grade 7.

Complex Task Outline

First, the students must measure the actual dimensions of the school and draw a scale drawing in order to build their models. You should provide inaccessible dimensions such as height.
Rather than providing the students with a scale factor, you will provide them with a limited surface area on which they will build. They will then choose their own ratio of similarity. Using the scale factor, they will find the dimensions of their model.

The students will then choose and have to purchase some of the available building materials as well as glue. They will be limited by a small budget provided by the admin and will have to plan their purchasing before they start construction, submitting an order form.

Each material will have a separate cost per pack (leaving the students the task of finding the unit rate). They will need to find the total area of their model, in order to determine whether or not their materials will bring them over budget. They will also be given dimensions of materials in order to calculate how many of each they will need.

Finally, the students will build their scale models! They will also write up a task report to demonstrate their procedure and outline the calculations and assumptions they used.

Notes for Teachers

This complex task is designed to be as realistic as possible, so rather than providing you with a set of manufactured constraints, we leave it up to you as the teacher to provide reasonable constraints given your situation. Some of these could be:

- Group size and composition: we suggest 3-4 students as a good balance.

- Available surface area for scale models: we suggest using the size of a school desk as the maximum area, and that students should be encouraged to make their models as large as possible within those constraints.
- We have provided a sample materials list along with prices and the sample student order form. You may use these as-is or adapt them for your own class.
- To go along with our suggested materials list and prices, we suggest a budget of $15 per project. This can be just tracked on the order sheet, or if you are feeling creative you could design some paper money for the students to buy supplies with.
- If your school is not composed of easily constructible shapes for grade 8 students, you can choose to simplify the task. For example, you may choose to instruct students to ignore the curved auditorium wall and approximate this with a straight side.

Keep in mind that the student groups are responsible for creating scale models with only the necessary intervention from yourself as the teacher. Your role will be to scaffold their progress as groups and to provide structured time for them to work on the project, rather than giving them all the information and steps to complete the task.

Finally, we suggest that to promote students to be motivated by the task, that you actually arrange with administration to showcase all the school models somewhere in the school.

Materials

While you can use your discretion and creativity, we suggest the following materials and prices:

- Popsicle sticks ($0.50 / pack of 25)
- Straws ($0.75 / pack of 50)
- Toothpicks ($0.50 / pack of 50)

It is appropriate to give each group a budget of $15 with these prices.

Materials should be distributed in packs so students practice finding a unit rate when they determine how many materials they need (or to make price comparisons!). You can of course change the package size as convenient.

Student Deliverables

The students are to produce two deliverables for the project: the **scale model** and the **task report**.

Scale model:

As the task overview outlines, student groups are to create a scale model of their school with the constraints (material and size) outlined above. Requirements for the scale model include:

- Size restriction (example: must fit on a student desk)

- Materials restriction: must be built with only materials supplied by the teacher and bought on budget
- Must as accurately as possible represent the school

Task report:

Student groups are also responsible for writing a report on their scale model. The requirements for this report should generally be as follows:

- Outline the group's procedure
- Outline how each member of the group contributed to the overall project
- Explain each mathematical step the group took (e.g. what the scale factor is, how the dimensions of the model were found)
- Answer reflection questions (sample questions in student handout)
- Be clearly organized and detailed

In the student handout attached to this teacher's guide, we have been more specific in our explanation of student deliverables. You should edit this handout if you plan to change the expectations of students.

Suggested Class Plan

We suggest that you run this complex task over several classes, interwoven with the Learning Activities in the LES. A possible class plan would be:

Class	*Activities*	*Required concepts*
1 – Intro and measurements	• Introduce the complex task to students, handing out the student guide and forming groups • Have groups measure the school and record their measurements on a floor plan	Solids (prior to LES)
2 – Planning the model	• Introduce students in depth to the size and material constraints of their model • Give groups time to come up with a scale factor and to start drawing a to-scale floor plan • Once the floor plan is complete, groups should decide on their materials and	Ratios, scale factors

	work out what materials they need to buy as well as their plan for building	
3 – Materials buying day!	• Turn the class into a marketplace for groups to submit order forms and buy materials • Allow time for groups to test out their materials and change their order if necessary. • Once materials are bought, groups should start building their models. You will probably want to keep the "store" open for groups who have miscalculated their material needs	Budgeting, unit rates
4 – Presenting the models	• Have groups present their finished products to class or admin • You may ask them to present some of their procedure, as outlined in their task report	All topics in LES covered

Groups will likely need time in between the four classes above to work on their plans, build their scale models and write their task reports. It is up to you whether you want to provide this as class time or have them work mainly outside of class.

Marking Rubric

This rubric is adapted from the MELS *Administration and Marking Guide*.

		Observable indicators				
		5	**4**	**3**	**2**	**1**
Evaluation Criteria	**Indication that the complex task has been understood** - Did the group complete the task? - Did they observe the set constraints?	The group: - Carries out all the steps to complete the task - Takes all the relevant information and constraints into account	The group: - Carries out all or most of the steps - Takes most of the relevant information and constraints into account	The group: - Carries out several steps - Takes several pieces of relevant information and constraints into account	The group: - Carries out some of the steps - Takes some of the relevant information and constraints into account	The group: - Begins to carry out some of the steps or carries out very few of the steps - Takes very little relevant information and few constraints into account
	Application of appropriate	- Uses the required mathe-	- Uses most of the required	- Uses several of the required	- Uses some of the required	- Uses very few of the required

	mathematical knowledge - Did the group apply the mathematical concepts of proportional situations correctly?	matical concepts and processes - Presents a correct solution or makes only a few minor mistakes (e.g. miscalculations, inaccuracies, omissions)	mathematical concepts and processes - Presents a solution or procedure containing few conceptual or procedural errors	mathematical concepts and processes - Presents a procedure containing some conceptual or procedural errors	mathematical concepts and processes - Presents an incomplete procedure that includes several conceptual or procedural errors	mathematical concepts and procedures - Presents an inappropriate or largely inappropriate procedure that includes several conceptual or procedural errors
	Development of appropriate procedure - Did the group develop an appropriate procedure? - Did the group work well together?	- Shows complete and organized work - Works well as a team to with equitable contributions from members	- Shows organized work, though some steps are implicit - Works as a team with roughly equitable contributions from members	- Shows somewhat unorganized work or leaves several steps implicit or missing - Works as a team most of the time, with contributions from all members	- Shows work consisting of confusing and isolated elements - Works as a team some of the time, with inequitable contributions from members	- Shows little work - Does not work as a team, or has very inequitable contributions from members
	Appropriate validation of the complex task *	- Validates their solution and rectifies it, if necessary	- Validates most of the steps in their procedure and rectifies it, if necessary	- Validates some of the steps in their procedure	- Reviews very few of their results	- Does not review their results

* The work involved in validating the solution may not always be fully shown. Students must be given feedback regarding this criterion, but it should not be taken into account in determining the student's mark.

Name: .... Group: .... Date: ....

Panorama 11

From Ratios to Similar Figures

Ratios and Rates

- A **ratio** is a type of **comparison** between two quantities or two magnitudes of the **same nature** expressed in the **same units** and involves the concept of **division.**
  The ratio a to b is written a:b or $\frac{a}{b}$, where $b \neq 0$.

Ex.: Julie has \$13 and Joey has \$9. The ratio of what Julie has to what Joey has is 13:9 or $\frac{13}{9}$.

- A **rate** is a type of **comparison** between two quantities or two magnitudes, generally of different natures, expressed in **different units,** and that involves the concept of **division.** Rates are written using a fraction bar. Expressed in words, rates generally involve words such as *in, for, per* and *each.*

Ex.: 315 g for 2 L or $\frac{315\text{ g}}{2\text{ L}}$

- When the denominator of a rate is 1, it is referred to as a **unit rate** and the 1 is omitted in the notation.

Ex.: 180 km/h

- If two ratios or two rates correspond to the **same quotient,** they are said to be **equivalent.**

Ex.: 1) The ratios $\frac{4}{5}$ and $\frac{8}{10}$ are equivalent since $\frac{4}{5} = 0.8$ and $\frac{8}{10} = 0.8$.

2) The rates $\frac{18\text{ g}}{5\text{ L}}$ and $\frac{54\text{ g}}{15\text{ L}}$ are equivalent since $\frac{18\text{ g}}{5\text{ L}} = 3.6$ g/L and $\frac{54\text{ g}}{15\text{ L}} = 3.6$ g/L.

- We can compare two ratios or two rates by bringing them to the same denominator or by calculating their quotients.

Ex.: $\frac{21}{5} > \frac{54}{15}$ since $\frac{63}{15} > \frac{54}{15}$ or $4.2 > 3.6$

$\frac{24\text{ g}}{15\text{ min}} < \frac{45\text{ g}}{25\text{ min}}$ since $\frac{120\text{ g}}{75\text{ min}} < \frac{135\text{ g}}{75\text{ min}}$ or 1.6 g/min < 1.8 g/min

1 Reduce the following ratios.

a) 5:15 [1:3] b) 12:48 [1:4] c) 51:34 [3:2] d) 84:16 [21:4]

e) $\frac{21}{15}$ [$\frac{7}{5}$] f) $\frac{72}{42}$ [$\frac{12}{7}$] g) $\frac{24}{144}$ [$\frac{1}{6}$] h) $\frac{14}{182}$ [$\frac{1}{13}$]

1s

Name: .... Group: .... Date: ....

2 In each case, compare the two ratios by using the appropriate symbol: <, > or =.

a) 7:12 > 19:36 b) 15:4 < 13:3 c) $\frac{41}{16} < \frac{52}{20}$ d) $\frac{23}{45} > \frac{4}{9}$

e) 3:5 < 22:35 f) 84:14 = 78:13 g) $\frac{66}{14} = \frac{99}{21}$ h) $\frac{16}{47} > \frac{7}{23}$

3 Emmy has three beakers. Beaker A contains 12 g of salt for 100 mL of water, Beaker B contains 19.5 g of salt for 150 mL of water, and beaker C contains 27.5 g of salt for 250 mL of water. Determine which beaker contains the saltiest solution.

Beaker A: 12 g for 100 mL is equivalent to 0.12 g/mL
Beaker B: 19.5 g for 150 mL is equivalent to 0.13 g/mL
Beaker C: 27.5 g for 250 mL is equivalent to 0.11 g/mL

Answer: Beaker B

4 Three friends run a distance of 2.2 km. Francis runs at a speed of 13 km/h, Ali maintains a speed of 4 m/s and Anthony runs the entire distance in 11 min. Who runs fastest?

Francis' speed: 13 km/h
Ali's speed: 4 m/s = 14 400 m/h = 14.4 km/h
Anthony's speed: 2.2 km in 11 minutes = 0.2 km/min = 12 km/h

Answer: Ali

5 Two plumbers compare their earnings based on their latest billings. Plumber A billed $148 for three and a half hours of work, including a $50 travel fee. Plumber B billed $110.25 for two hours and fifteen minutes of work. A travel fee of $45 is included in his bill. Calculate each plumber's hourly rate.

Plumber A's hourly rate: 148 – 50 = 98; 98 ÷ 3.5 = 28
Plumber B's hourly rate:
110.25 – 45 = 65.25; 65.25 ÷ 2.25 = 29

Answer: Plumber A: $28/h Plumber B: $29/h

6 The initial resolution of a rectangular computer screen is 1152 by 864 pixels. The other resolutions available for the screen are 640 by 480 pixels, 800 by 600 pixels and 1024 by 768 pixels. After determining the ratio between the number of pixels in length by the number of pixels in height for each resolution, state if they are equivalent.

Initial resolution of 1152 by 864, 1152:864 = 4:3
Resolution of 640 by 480, 640:480 = 4:3
Resolution of 800 by 600, 800:600 = 4:3
Resolution of 1024 by 768, 1024:768 = 4:3

Answer: All the resolutions are equivalent.

Name: .... Group: .... Date: ....

7 Juan prepares a meal. He marinates some meat for 45 minutes and cooks it for fifteen minutes. Preparing the vegetables requires 10 min, while cooking them requires 20 min. Determine the ratio in each case.

a) $\frac{\text{meat's marinating time}}{\text{meat's cooking time}}$ $\frac{45}{15} = \frac{3}{1}$

b) $\frac{\text{vegetables' preparation time}}{\text{vegetables' cooking time}}$ $\frac{10}{20} = \frac{1}{2}$

c) $\frac{\text{meat's cooking time}}{\text{vegetables' cooking time}}$ $\frac{15}{20} = \frac{3}{4}$

d) $\frac{\text{meat's marinating time}}{\text{vegetables' preparation time}}$ $\frac{45}{10} = \frac{9}{2}$

8 In each case, determine the value that allows you to create equivalent ratios.

a) 8:9 = 16: 18

b) 6:28 = 9 :42

c) 12:46 = 30 :115

d) 23:14 = 69: 42

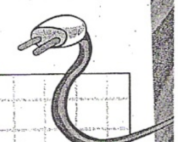
e) $\frac{6}{14} = \frac{3}{7}$

f) $\frac{8}{26} = \frac{12}{39}$

g) $\frac{14}{21} = \frac{34}{51}$

h) $\frac{18}{24} = \frac{120}{160}$

9 Three friends compare their electricity bill. John, who lives in Ontario, paid $125 for 2500 kW. Jennifer, who lives in British Columbia, paid $187.55 for 3100 kW. For his part, Jean-François, who lives in Québec, paid $140.56 for 2800 kW. What is the unit rate paid by each person?



Answer: John's unit rate: 5 ¢/kW. Jennifer's unit rate: 6.05 ¢/kW.
Jean-François' unit rate: 5.02 ¢/kW.

10 In music, the value of a whole note is twice that of a half note, and a half note is twice as long as a quarter note. In addition, an eighth note is as long as half a quarter note. Determine the ratio for each case.

a) $\frac{\text{value of a whole note}}{\text{value of a half note}}$ $\frac{2}{1}$

b) $\frac{\text{value of a quarter note}}{\text{value of a whole note}}$ $\frac{1}{4}$

c) $\frac{\text{value of an eighth note}}{\text{value of a quarter note}}$ $\frac{1}{2}$

d) $\frac{\text{value of a whole note}}{\text{value of an eighth note}}$ $\frac{8}{1}$

11 In a co-ed classroom of 32 students, there are 14 girls. In each case, determine the reduced ratio.

a) $\frac{\text{number of girls}}{\text{number of boys}}$ $\frac{7}{9}$

b) $\frac{\text{number of girls}}{\text{students in the class}}$ $\frac{7}{16}$

c) $\frac{\text{students in the class}}{\text{number of boys}}$ $\frac{16}{9}$

d) $\frac{\text{students in the class}}{\text{(number of boys) + (number of girls)}}$ $\frac{1}{1}$

Name: .... Group: .... Date: ....

12 Here are the ingredients for a crepe recipe. Determine the quantity of ingredients for one portion.

Crepes

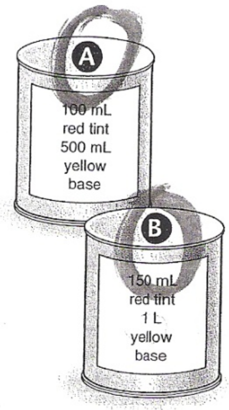
For 4 portions

- 2 cups of flour
- $1\frac{1}{2}$ cups of milk
- 3 eggs

Answer:

$\frac{1}{2}$ cup of flour, $\frac{3}{8}$ cup of milk and $\frac{3}{4}$ of an egg

13 Here are the colour combinations for two containers of orange paint. Determine which container will result in a darker hue.



Combination Ⓐ: $\frac{\text{quantity of red tint}}{\text{quantity of yellow base}} = \frac{150}{1000} = \frac{3}{20}$

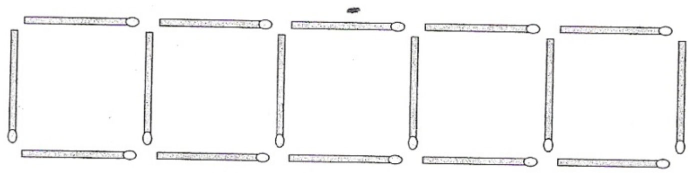
Combination Ⓑ: $\frac{\text{quantity of red tint}}{\text{quantity of yellow base}} = \frac{100}{500} = \frac{4}{20}$

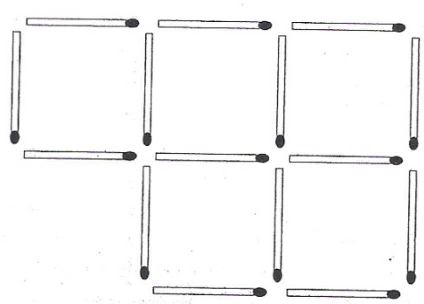
Answer:

Combination Ⓑ has the darker hue.

Challenge

14 The boundary of the pattern below is made up of 12 matchsticks, where $\frac{\text{total number of matchsticks}}{\text{number of small congruent squares}} = \frac{16}{5}$. Create a pattern whose boundary is made up of ~~10~~ matchsticks and where $\frac{\text{total number of matchsticks}}{\text{number of small congruent squares}} = \frac{15}{5}$.





Other handouts (with solution key):

1 A restaurant cook prepares his recipe for spaghetti sauce. He usually adds 15 mL of salt for flavour.

The following week, he decides to adjust his original recipe to make it less salty.

Which adjustments should the cook make to ensure that his sauce is less salty?

A) Add a can of tomato juice.

B) Reduce the amount of tomato juice by one can.

C) Double the recipe.

D) Prepare half the recipe.

1 ANS A

2 A photograph measured 15 cm by 20 cm. Peter enlarged the photograph. It is now 50 cm long.

What proportion can be used to find the width of the enlarged photo?

The proportion is ______________________________.

2ANS The proportion is $\frac{15}{20} = \frac{x}{50}$ or an equivalent proportion.

3

In a high school, 75 % or 168 of the girls are 15 years old or older.

The ratio between the number of girls and the number of boys in the school is 14 : 11.

How many students are there in this school?

Show your work.

3ANS

Example of an appropriate solution

Number of girls in the school

$$\frac{75}{100} = \frac{168}{x}$$

$$x = \frac{168 \times 100}{75} = 224$$

Number of boys in the school

$$\frac{14}{11} = \frac{224}{y}$$

$$y = \frac{11 \times 224}{14} = 176$$

Number of students in the school

224 + 176 = 400

Answer There are 400 students in this school.

4 Canadian athletes won 13 medals at the 1994 winter Olympics in Lillehammer.

The types of medals won are listed in the table below.

Canada's Medals at Lillehammer

Gold	Silver	Bronze	Total
3	6	4	13

Quebec athletes won 9 of these medals. Myriam Bédard won two gold medals and a silver medal.

Express as a ratio the number of medals won by Myriam Bédard to the total number of medals won by Canadian athletes.

The ratio is _______________.

4ANS The ratio is $\frac{3}{13}$ or 3:13.

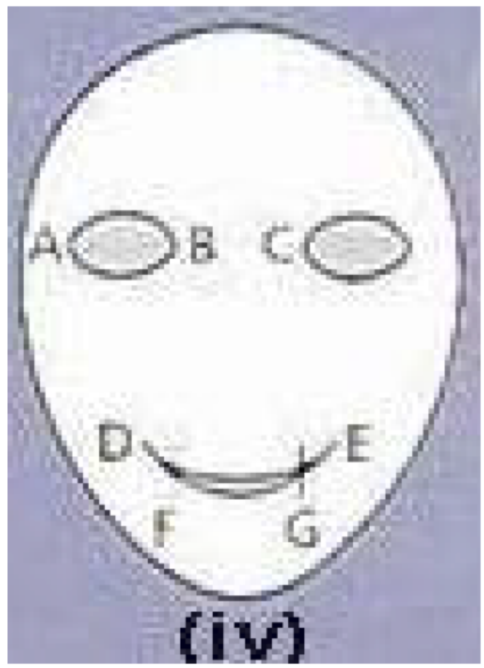
Facial Ratios

Source for activity instructions: http://www.sycd.co.uk/who_am_i/pdf/passport/gmpupils.pdf

You are going to find out just how different people's faces really are. You are going to look at these measurements:

- the width of your picture's eyes
- the width of the bridge of your picture's nose
- the width of the whole smile
- the width of teeth

Stick a photo in the space provided. It needs to show the front of the face



Measure the width of one eye (from A to B on the diagram).
Eye width = ________________ mm
Measure the width of the bridge of your nose (B to C).
Bridge width = ________________ mm
Measure the width of your mouth when you smile (D to E).
Mouth width = ________________ mm
Measure the width of your teeth when you smile (F to G).
Teeth width = ________________ mm

You are going to work out the ratios between some of your measurements by dividing them. You do this by dividing one measurement by another. Ratios are a good bit of maths to help us compare different numbers. You're going to use them for this activity.

$$\frac{\text{bridge width}}{\text{eye width}} = \underline{\qquad\qquad\qquad} = \boxed{\qquad\qquad\qquad}$$

$$\frac{\text{mouth width}}{\text{teeth width}} = \underline{\qquad\qquad\qquad} = \boxed{\qquad\qquad\qquad}$$

• Are the two ratios the same as each other?

• Are they close to each other?

Comparing faces

- Now you are going to see how your ratios compare with other peoples' pictures in your class.
- Write down all the widths and ratios for ten people's pictures in your class.
- Work out the average ratio for these ten people. (To work out the average add up all ten values and divide the total by ten.)

Name	Bridge width (mm)	Eye width (mm)	Bridge/ eye ratio	Mouth width (mm)	Teeth width (mm)	Mouth/ teeth ratio
Totals						
Averages						

Were the ten ratios for bridge/eyes different from each other?

• Were they very different? If the measurements are not exact then the ratios may be a little different.

• Now look at the mouth/teeth ratios.

What is the average for this ratio?

• Are the two ratios from the same person similar?

The Ancient Greeks knew a lot about these ratios. The Greeks took lots of measurements of faces, bodies and other parts of living things. They found that the same ratio kept appearing. They called it the Golden Ratio. They thought that if something matched the Golden Ratio this was a way of making sure it looked right. The Ancient Greeks used the Golden Ratio in the design of some of their buildings. This idea has been passed on, and there are some famous old buildings where the Golden Ratio was used.

Source for activity instructions: http://www.sycd.co.uk/who_am_i/pdf/passport/gmpupils.pdf

PART 2a: Rates

1) Look at the statements below:

Miles to gallons **pizza to people**

Dollars to hours worked **grapes to pounds**

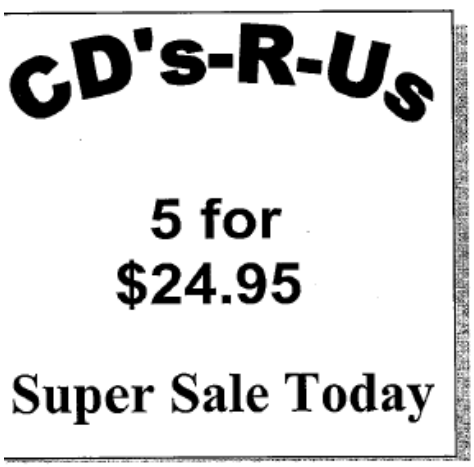
Miles per hour **people to square miles**

a.) What do you notice about each statement? William looked at these statements and said that they were all a comparison of two things. Write another comparison of two things.

This type of comparison is called a **RATE**.

Part 2b: Using Unit Rates

The advertisements below use rates to describe some sale prices:



Which sale is the better buy? Explain your thinking?

2) George said that he used estimation and division to find the UNIT RATE. He used division to find out how much ONE disc cost for each sale so he could compare the price equally.

a.) If one sale said that grapes were 4 pounds for $2.99 and another sale was 5 pounds for $4.40. Which is the better buy? What is the UNIT RATE for each sale (price per pound)?

b.) Juan went on a trip that was 458 miles. He used 15 gallons of gas. What was his mileage (miles per gallon of gas)?

c.) At one gas station on his trip he paid $31.00 for 14 gallons. He noticed that the receipt didn't have the price per gallon (pump price). What was the price of a gallon of gas?

3) George and Juan compared the fuel economy of their cars and found these rates:

George's car went 580 miles on 20 gallons of gas.
Juan's car went 450 miles on 15 gallons of gas.

a.) Compare the mileage (unit rate – miles per ONE gallon of gas).

b.) How far can George travel on 2 gallons of gas? On 5 gallons of gas? On 20 gallons of gas?

4) Tonya used a RATIO table to solve another problem. This is what her table looked like to find out the mileage for George's car.

Gallons of Gas	0	1	2	3	4	5	6	7	8
Miles Traveled									

a.) Complete the table to find the miles traveled.

b.) Look at the patterns in your table. Write an equation for a rule you could use to predict the miles driven (m) from the gallons of gas used (g).

c.) Use the rule you wrote for part 4b to find how far George's car would travel on 9.5 gallons? 19 gallons? 24gallons? 100 gallons? 125 gallons?

5) Crystal decided to use the ratio table for some of her computations. She knew that she could do this because rates are ratios.

Gallons	1	2	10	5	15
Miles	22	44	220	110	330
Action					

a.) Complete the last row of the table to explain how Crystal found the answers in the columns.

b.) Make a ratio table to find the miles traveled on 19 gallons, 24 gallons, 100 gallons, and 125 gallons.

Gallons	1							
Miles	22							
Action								

c.) Bonus: How could you find the miles traveled on 0.5 gallons using the ratio table? Use the ratio table to show how you would calculate the miles traveled on 9.5 gallons of gas.

Gallons								
Miles								
Action								

Answers:

PART 2a: Rates

1) Look at the statements below:

Miles to gallons **pizza to people**

Dollars to hours worked **grapes to pounds**

Miles per hour **people to square miles**

a.) What do you notice about each statement? William looked at these statements and said that they were all a comparison of two things. Write another comparison of two things. **They compare one thing to another thing. When I saw the PER in the "miles per hour" I knew I would have to compare the number of miles I can drive in ONE hour. The pizza to people reminds me of when I have friends over and I have to figure out how many pizzas to order because I need to think about how much pizza EACH person will eat. My mom said that many people get paid by the hour.**

This type of comparison is called a **RATE.**

Part 2b: Using Unit Rates

The advertisements below use rates to describe some sale prices.

CD's-R-Us

5 for
$24.95

Super Sale Today

1.) Which sale is the better buy?
CD-R-US will be the better buy because it is less money per CD.
Explain your thinking?
CD-R-US: 24.95/5 = 4.99 each (This looks like a fraction, but it means division.) CD-Place: 36.95/7 = 5.28 each

2.) George said that he used estimation and division to find the UNIT RATE. He used division to find out how much ONE disc cost for each sale so he could compare the price equally.

a.) If one sale said that grapes were 4 pounds for $2.99 and another sale was 5 pounds for $4.40. Which is the better buy? What is the UNIT RATE for each sale (price per pound)? **Better buy: 4 for $2.99 This is a unit rate of approximately 75 cents per pound of grapes. The other sale costs 88 cents per pound of grapes.**

b.) Juan went on a trip that was 458 miles. He used 15 gallons of gas. What was his mileage (miles per gallon of gas)? **458/15 = 30.53 miles per gallon**

c.) At one gas station on his trip he paid $31.00 for 14 gallons. He noticed that the receipt didn't have the price per gallon (pump price). What was the price of a gallon of gas?
$31.00/14 = $2.21 per gallon

I first used estimation to get a rounded answer. I thought $30 divided by 15 is $2 per gallon. There is $1 more money and 1 less gallon of gasoline than my estimation, so the actual answer will be a little over $2.00.

3) George and Juan compared the fuel economy of their cars and found these rates:

George's car went 580 miles on 20 gallons of gas.
Juan's car went 450 miles on 15 gallons of gas.

a.) Compare the mileage (unit rate – miles per ONE gallon of gas).
George's unit rate is 580/20 = 29 miles per gallon. Juan's unit rate is 450/15 = 30 miles per gallon.

b.) How far can George travel on 2 gallons of gas? **29 x 2 = 58 miles** On 5 gallons of gas? **29 x 5 = 145 miles** On 20 gallons of gas? **29 x 20 = 580 miles**

4) Tonya used a RATIO table to solve another problem. This is what her table looked like to find out the mileage for George's car.

Gallons of Gas	0	1	2	3	4	5	6	7	8
Miles Traveled	**0**	**29**	**58**	**87**	**116**	**145**	**174**	**203**	**232**

I added 29 miles each time. The miles traveled increased by 29 for each gallon of gas. Sometimes I would do the math in my head. I looked at the previous miles, added 30 miles, and then subtracted 1 mile because 30 = 29-1.

a.) Complete the table to find the miles traveled.

b.) Look at the patterns in your table. Write an equation for a rule you could use to predict the miles driven (m) from the gallons of gas used (g).
m = 29g

c.) Use the rule you wrote for part 4b to find how far George's car would travel on 9.5 gallons? **275.5 miles** 19 gallons? **551 miles** 24gallons? **696 miles** 100 gallons? **2900 miles** 125 gallons? **3625 miles**

5) Crystal decided to use the ratio table for some of her computations. She knew that she could do this because rates are ratios.

Gallons	1	2	10	5	15
Miles	22	44	220	110	330
Action	**1 x 22**	**2 x 22 (doubled)**	**10 x 22**	**5 x 22 OR Took ½ of 220 (5 is ½ of 10)**	**15 x 22 OR Added 220 (10x22) and 110 (5x22) = 330**

a.) Complete the last row of the table to explain how Crystal found the answers in the columns.

b.) Make a ratio table to find the miles traveled on 19 gallons, 24 gallons, 100 gallons, and 125 gallons.

Gallons	1	**5**	**10**	**20**	**19**	**25**
Miles	22	**110**	**220**	**440**	**418**	**550**
Action	**1 x 22**	**5 x 22**	**10 x 22 OR 2 x 110**	**2 x 220 OR 20 x 22**	**19 x 22 OR 440 -22**	**110 + 440 (20 gal. + 5 gal.)**

Gallons	**24**	**50**	**100**	**105**	**125**
Miles	**528**	**1100**	**2200**	**2310**	**2750**
Action	**550 mi. – 22 mi. (24 gal. is 25 gal. minus 1 gal.)**	**110 x 10 (110 mi. x 10 equals 5 gal. x 10)**	**220 x 10 (220 mi. x 10 equals 10 gal. x 10)**	**105 x 22 OR 100 gal + 5 gal equals 2200 miles + 110 miles**	**125 x 22 OR 100 gal. + 25 gal. equals 2220 miles + 550 miles**

I used the answers in some columns to help with the answers in other columns. For example, I found 25 gallons by adding together the gallons for 20 and 5. I found 100 gallons, by multiplying that answer by 4. I found 19 gallons by finding 20 gallons and subtracting 1 gallon. To find out how many miles it took for 125 gallons of gas, I looked at the table and added together the miles for 100 gallons and 25 gallons (2200 miles + 550 miles).

c.) Bonus: How could you find the miles traveled on 0.5 gallon using the ratio table? Use the ratio table to show how you would calculate the miles traveled on 9.5 gallons of gas.

Gallons	**.5**	**1**	**1.5**	**2**	**2.5**	**8.5**	**9**	**9.5**
Miles	**11**	**22**	**33**	**44**	**55**	**187**	**198**	**209**
Action	**22/2**	**1 x 22**	**11 + 22**	**22 +22**	**44 +11**	**8 x 22 + 11**	**9 x 22**	**198 +11**
	Half of 1 gal.		**Add 1 gal. and ½ gal.**	**doubled**	**Add 2 gal. and ½ gal.**			

Another answer:

1) **I knew 1 gal. was 22 1:22 (gallons : miles).**
2) **To find 10 gallons, I multiplied by 10 (10:220).**
3) **To find 0.5 gallons -- I knew this was ½ of 1 gallon (0.5:11).**
4) **I knew 9.5 gallons equaled 10 gallons minus 0.5 gallons.**
5) **I used the ratios and calculated 220 miles – 11 miles in my head. First I subtracted 10 and then I subtracted 1. (9.5 gallons : 209 miles)**

SUPPLEMENTARY MATERIAL:

1 Christopher went to the supermarket to buy some olives. He found that they are sold in 750 mL jars for \$3.29 and also in 1.5 litre jars for \$5.99.

How much will Christopher save if he buys 3 litres of olives in the more economical format?

Show your work.

1ANS Work : (example)

<u>case 1</u> :750 mL jar at \$3.29 each

3 litres = 3000 mL

= 4×750 mL

Cost of four 750 mL jars : $4 \times 3.29 = 13.16$

<u>Case 2</u> : 1.5 litre jars at \$5.99 each

3 litres = 2×1.5 litre

Cost of two 1.5 litre jars : $2 \times 5.99 = 11.98$

Amount saved : $13.16 - 11.98 = 1.18$

Result Christopher will save \$1.18.

<u>2nd class:</u>

Fill in the following chart with a list of the same items from your shopping cart from different stores in your neighborhood and see which store offers the better deal. Calculate the rate associated with each item and how much it costs per one item (if applicable).

Store 1:	Store 2:

How much do you save if you were to take the more economical buy?

Savings:

____

____

____

____

Monthly incomes/spendings (you are encouraged to use a bulletin board (poster) to post your data/calculations. This will be graded on creativity and accuracy.):

____

____

____

____

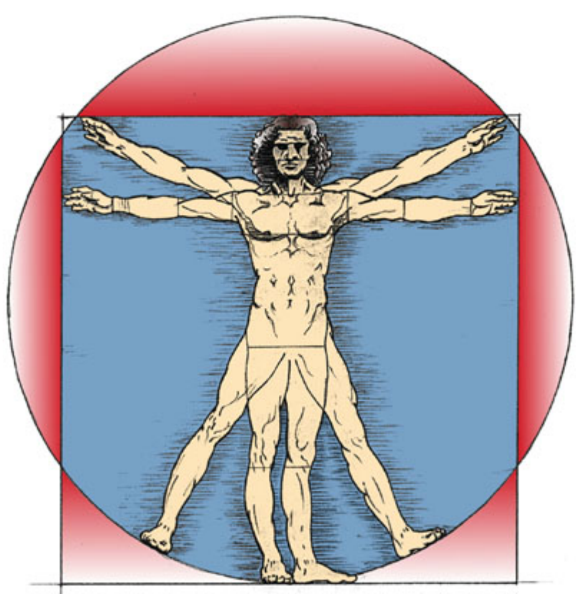
The Vitruvian Man - Proportions Activity

Vitruvius, a Renaissance architect, described the dimensions of the human body. They are listed below. Artist, scientist and philosopher, Leonardo Di Vinci, illustrated his theory in the year 1490.

Your task is to test Vitruvius' theories. You need to propose a hypothesis, design a procedure, collect (repeated trials), analyze, and discuss your data (and any possible sources of error), and draw conclusions. Finally, you will share your information with the class.

<u>Vitruvius's Theories</u>

- From fingertip to fingertip, the span of a person's arms is equal to his/her height.
- From the roots of the hair to the bottom of the chin is the tenth of a person's height
- From the bottom of the chin to the top of the head is one eighth of a person's height
- The distance from the bottom of the chin to the nose and from the roots of the hair to the eyebrows should be equal, each comprising 1/3 the length of the face.
- From the bottom of the knee to the bottom of the foot is equal to ¼ of a person's height.
- The distance from the outer edge of one shoulder to the outer edge of the opposite shoulder is equal to ¼ of a person's height.



Name: _______________________________ Math 212

DIRECT PROPORTIONALITY

A situation that has equal ratios or rates is a ______________________________________.

In a table of values that represents a **direct proportional situation**, *the number in the first row or column and the number in the second row* or column from a proportional series of numbers.

In other words, they are said to be **proportional** because the values in the **first row** or column are **multiplied by** a number called ________________________________

Example:

The following table represents the rental cost from an office supply company.

a) What is the **constant of proportionality** for this situation?

.

Time (hours)	0	1	2	5	8
Cost ($)	0	2.50	5	12.50	20

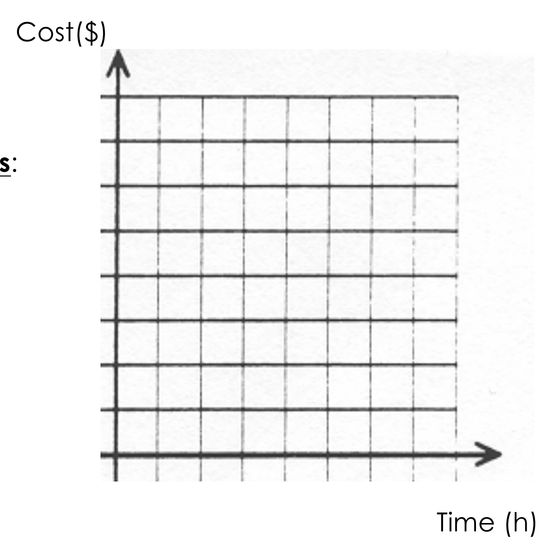
b) Calculate the rate of change for the values in the table. ____________________

c) What is the y-intercept? ______________________

d) What is the rule for this situation? ______________________

e) In general, the rule for any direct proportional situation is: ___________________

f) Graph this situation.



g) In general, a direct proportional situation has a graph with the following **characteristics**:

- They will then be give the second part, and have the rest of the class to work in pairs, and solve the following problems (any incomplete problems are to be done for homework):

PRACTICE

Example 1: Are the following situations proportional? If so, why?

a)

Mass (kg)	0.5	1	2	5
Cost ($)	2	4	8	20

b)

Age	2	4	6	8
Height (cm)	90	98	112	130

Example 2: A supermarket has a big sale on turkeys just before Thanksgiving. The discount price is $2.40 a kilogram.

a) Complete the following table of values.

Mass (kg)	5	6	7	8	10
Cost ($)					

b) What is the constant of proportionality? __________

c) What is the rule? ___________________

Example 3 :

Jonathan makes a fixed salary of 400$/week. If he works overtime, he will make an extra 15$ and hour for every additional hour of work.

Jonathan's Weekly Salary

a) Make a table of values for this situation.

b) Does this table represent a proportional situation? Show why or why not? ________

c) What is the unit rate? _______________

d) Write the equation/rule _______________

INVERSE PROPORTIONALITY

Every year, the Secondary 5 students participate in the "Student Stock Exchange". Last year, "Raise the Steaks" made a $2000 profit. They were told that they had to equally divide their profit among all their student shareholders.

Number of share holders	
Student Profit	

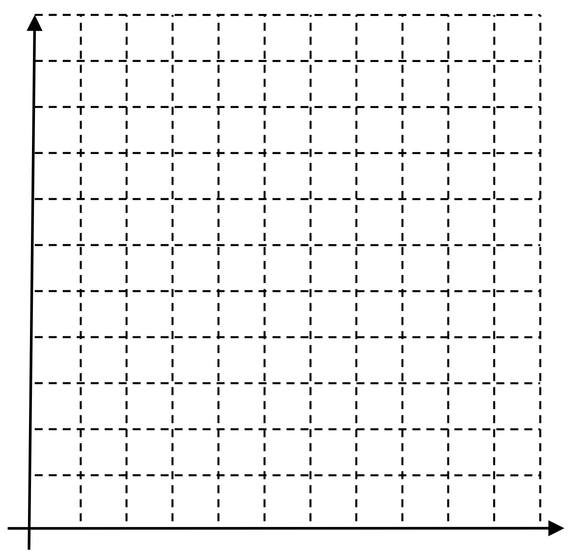
a) Construct a table of values for this relation.

Rule:__________

b) What is the pattern among the two variables in questions?

c) Complete the Cartesian graph.

Student Profit

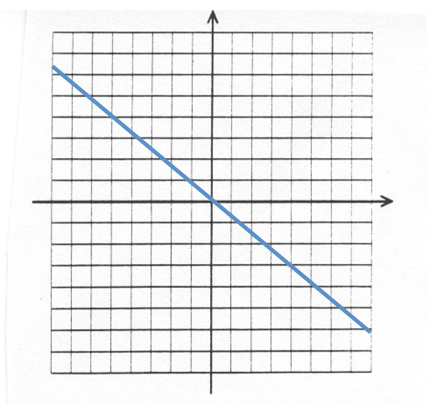


Characteristics:

Take a look!

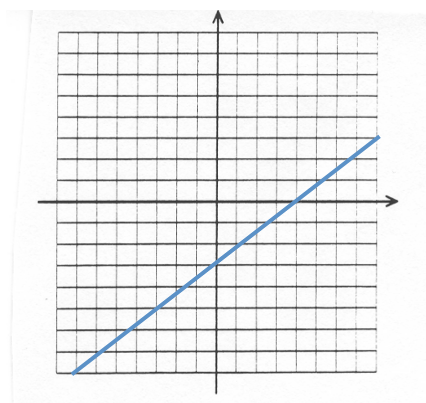
1) State if the following graph represents a direct proportional situation, an inverse proportional situation or neither.

a)



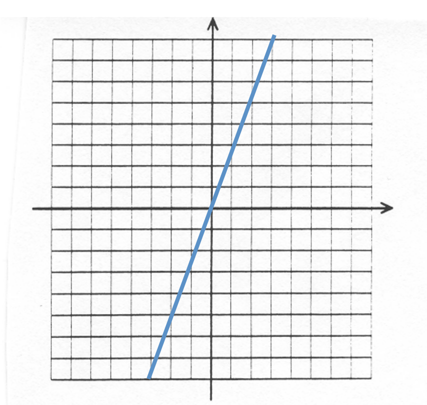
____

b)

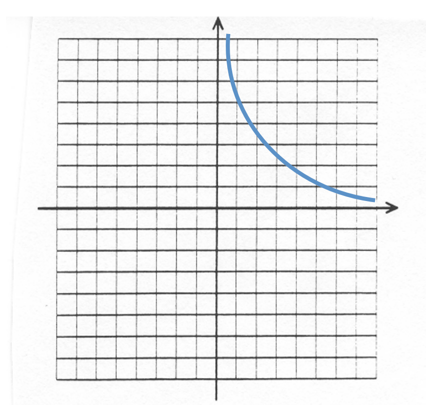


____

c)



d)



2) State if the following equations represent a direct proportional situation, an inverse proportional situation or neither.

a) $y = -5x$ b) $y = 3x - 2$ c) $y = x^2$ d) $y = \frac{250}{x}$

3) State if the following tables represent a direct proportional situation, an inverse proportional situation or neither.

a)

X	Y
0	1
2	11
4	21
6	31

b)

X	Y
2	-14
3	-21
6	-42
10	-70

c)

X	Y
2	12.5
5	5
10	2.5
12.5	2

Proportions Quiz

<u>When applicable, show your work and round your answers to two decimals.</u>

1. For each of the following **find the missing term.** **(2 marks each)**
(Show your work / Use any method of your choice.)

a) Mr Burns ran 6 km in 30 minutes. How far would he run in 54 minutes?

Answer:________________

b) A Kia Sol can drive 100 km on 7 litres of gas. How far would you be able to drive on 42 litres of gas in the same car?

Answer:________________

c) Wendy's heart beats 14 times in *12 seconds*. How many times does it beat in *2 hours*?

Answer:________________

d) The ratio of **girls to boys** in "Elmo's Elementary School" is **3:5**. If there are 384 students in the school, how many **girls** are there?

Answer:________________

2- During the summer, campers at "Camp Campanelli" can rent sailboards for $5 an hour plus a $20 insurance deposit. **Is this a proportional situation?** (Explain/Prove your answer.)

3- Write if the following situations, tables and equations are either **Directly Proportional (DP), Inverse Proportional (IP) or Not Proportional (NP).**

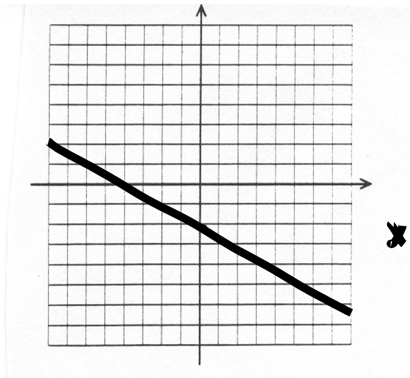
a) My rule is: $\mathbf{y} = \dfrac{\mathbf{55}}{\mathbf{x}}$. _______________

b) Larry fills his bathtub with 100 litres of water. When he pulls the plug, the water drains at a rate of $3L/min$. _______________

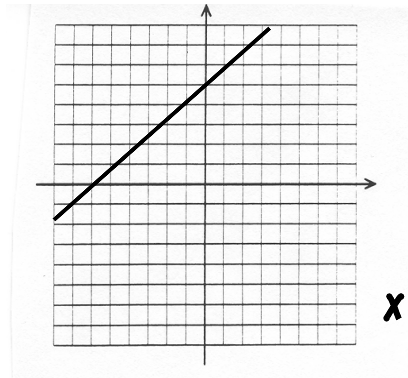
y

y

c)



d)



______________ ______________

e)

x	1	2	6	7.5
y	5	10	30	37.5

f)

x	1	2	2.5	5
y	10	5	4	2

______________ ______________

g) My rule is: **y = 50 - 2x** . ______________

4- The distance separating two towns is represented on a map with a **1: 150 000 scale**.

If the **actual distance** separating these two towns is **7.5 km**, what is the **observed distance on the map** between these two towns? **(Answer in cm.)** (2 marks)

Proportions Review

Short Answer Questions: *Answer in the space provided.*

1. Find the **unit rate**. (2 marks)

   a) She traveled 500 km on 12.5 liters of gas. **Unit Rate:** _________________

   b) He covered 37 m^2 of wall surface area with 4 liters of paint. **Unit Rate:** _______________

2. Match the definition with the appropriate phrase. (Place the letter on the line.) (3 marks)

PHRASE	DEFINITION
Directly Proportional Situation ____	**A)** An equality between 2 rates or ratios.
Coefficient of Proportionality ____	**B)** A comparison of 2 unlike quantities.
Ratio ____	**C)** The product of the two variables is a constant.
Rate ____	**D)** The quotients of ALL my corresponding values are equal.

3. Indicate whether the following situations are usually directly proportional situations (**DP**), inversely proportional situations (**IP**) or not proportional (**NOT**). (2 marks)

   a) The number of hours driving at a constant speed and the distance traveled. __________

   b) The cost of a pair of pants and the size of the pants. __________

   c) Michael throws a party and pays 550$ for the hall, he charges each person an equal share of the price of the hall. ____________

d) Anthony sells cars. He makes a weekly base salary of $250 and 3% commission per car that he sells. _____________

4. Determine if the following situations are examples of **ratios** or of **rates**. (3 marks)

a) The number of rainy days to the number of sunny days: ____________________

b) The distance a car is traveling and the time it takes to get somewhere: ________________

c) The amount of snow that falls during each month of winter: ______________

5. Explain the meaning of the following sentence: (1 mark)

A map indicates a scale of 1: 2000.

____

____

6. Complete the following tables of values **and** write the equation/rule. (3 marks)

a) Directly Proportional

X	Y
-2	6
0	
4	-12
	-33

b) Inversely Proportional

X	Y
4	
5	20
	10
20	5

Equation: ______________

Equation:_______________

7. Use the same units of measure to write ratios for the following:

(Reduce your answer.) (3 marks)

a) A nickel compared to a toonie: ________________

b) 400 seconds compared to three hours: ___________________

c) 2 decameters to 6 meters: ________________

Development Questions: Make sure to show your work and units! (2 marks each)

8. A jet travels at a velocity of **1250 km/hr.**

How many hours would it take for the jet to travel **5000 km?**

Answer: ______________

9. Mr Schnitzer was speeding in his Smart Car ® and got pulled over by the police. He was told by the officer that he was travelling at a speed of **125 km/hour.**

What was his speed in **m/sec?**

Answer: ___________

10. Jeremy is a training to become a professional boxer. At the moment, he has a record of **winning 2** boxing matches **for every 7 he loses.** If he has competed in a **total of 54 matches** during the last year, how many did he **win?**

Answer: _______________

11. David and Liam run on the same track. If **David takes 55 minutes to run 16 laps** and **Liam can run 13 laps in 43 minutes,** who is **faster?**

Answer: _________________ **is faster.**

12. The Cities of Jackson, Mississippi, and Carson City Nevada, are 1 750 km apart. A map of The U.S.A. has a scale of 3 cm : 250 km. How far apart **on the map** are the cities (in cm)?

Answer: _______________

13. **Identify** and circle if the following tables or graphs represent Directly Proportional (**DP**), Inversely Proportional (**IP**) or Not Proportional (**NOT**) situations **and explain why** you made this choice. (6 marks)

a)

x	y
2	14
4	28
6	42

DP **IP** **NOT**

Why?________________________

____

____

b)

x	y
0	5
1	10
2	15

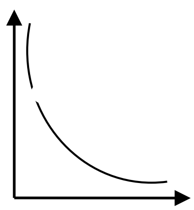
DP **IP** **NOT**

Why?_____________________

____

____

c)



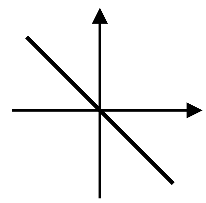
DP **IP** **NOT**

Why?________________________

____

____

d)



DP **IP** **NOT**

Why?____________________

____

____

14. The cost of a yearly membership at “Previewed Video” store is 25$. Once you have paid your membership fee, it costs 3.50$ for each video that you rent. (4 marks)

a) Using the given values, make a table representing your yearly cost.

# of videos rented	0	2	5	12	15	20
Total Cost ($)						

b) What is the **equation** representing this situation? ______________________________

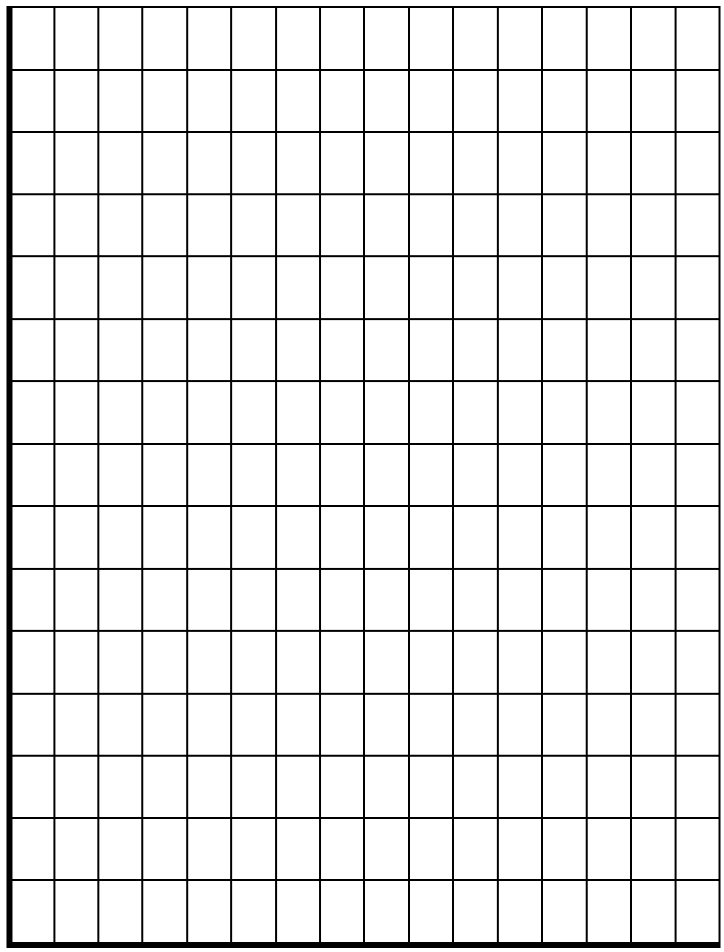
c) Is this a directly proportional situation? ______Why? ___________________________

____

d) Is this an inversely proportional situation? ______Why? _________________________

____

e) Using the Cartesian plane below, graph this situation. (3 marks)



15. John, Glenda, Stephane and Sophia each have a sum of money. The ratio of the money for John and Stephane is 8:12 and for Glenda and Sophia the ratio is 9:3. The ratio for John and Sophia is 10:15. If Stephane has $48**, find how much Glenda has.**

SIMILAR FIGURES

Two figures are **similar** when :

-____

-____

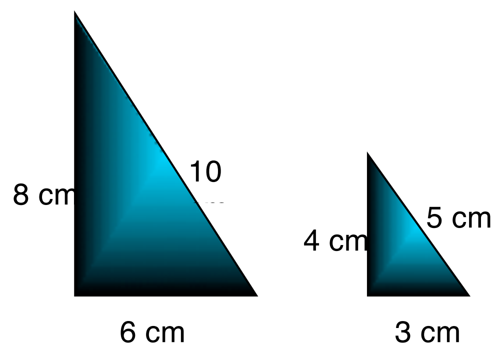
<u>Notes</u> :

-Similar figures are therefore **proportional situations**. (I.e. equal ___________ or rates.)

-The _________________________________ represents the number of times that an image is

bigger/smaller than it's initial figure.

<u>For example</u> :

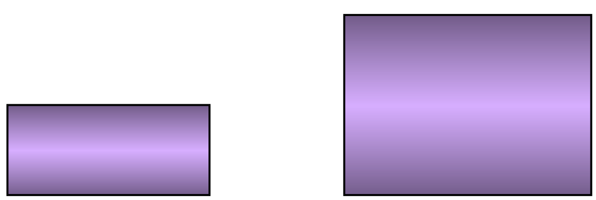


Practice

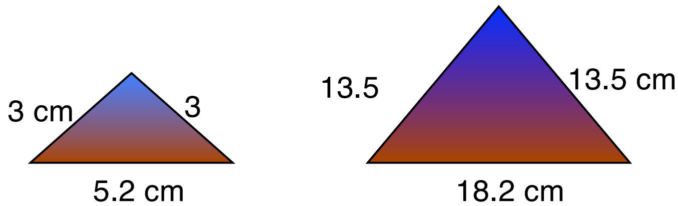
1) Are the following figures similar?

If yes, indicate the scale factor. If no, indicate why not.

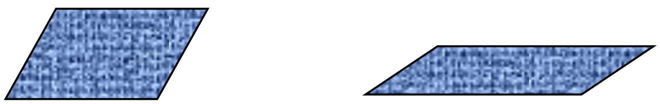
a)



b)

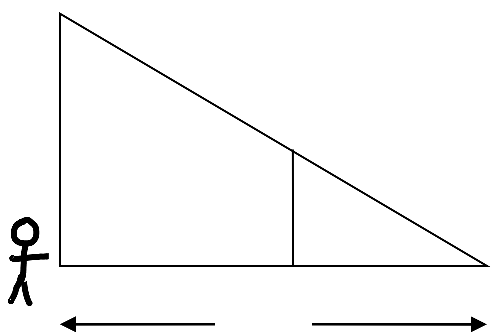


c)



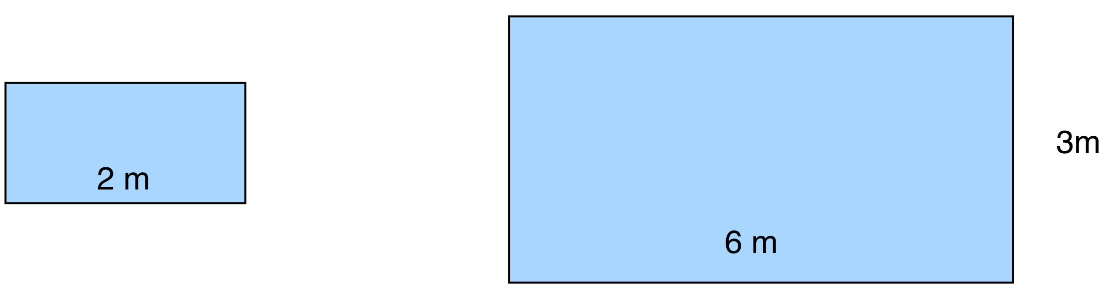
2) The illustration below represent situations that give rise to similar triangles.

Find the missing measure for each example.



Perimeters and Area of Similar Figures

Refer to the following similar figures and find the ratio of the perimeters and that of the areas.



What do you notice?

If "h" represents the similarity ratio/scale factor of two similar figures,

-The ratio of the perimeters will be equal to ________________

-The ratio of the areas will be equal to ________________

Building a Scale Model of the School
Student Guide

Task description

The school administration has asked your class to build a scale model of the school to display in the lobby and library! You will be learning about proportional situations while you collect measurements, draw a scale diagram, collect and budget your materials and finally create the model!

You will be responsible for:

- Measuring the school and recording your measurements,
- Finding an appropriate scale factor for your model,
- Creating a plan and budget for your model,
- Ordering the necessary materials from your teacher,
- Building the model, and
- Writing a report on how your group worked to complete the task.

Your teacher will give you more details on the timeline and steps.

Deliverables (what you need to hand in)

Scale model:
Your group is responsible for creating a scale model of the school using the materials provided by the teacher. The requirements are:

- Your model must fit onto your classroom desk and must take up most of the desk space, but other than that you can determine the size.
- You must buy your materials from your teacher with the order form on the back of this page. You have a total of $15.
- You must build solid walls for your model.

Task report:
You are also responsible for writing a report on how you built your scale model. The reports should be clearly organized and detailed, and should be at least 3 pages long. Your report should have four sections:

1. Procedure: outline the steps you took to build your model.
2. Teamwork: show how each group member contributed to the overall project.
3. Mathematical explanation: explain each mathematical step the group took (e.g. what the scale factor is, how the dimensions of the model were found).
4. Reflection questions: answer the following questions
    a. How did your group use the concept of *scale factor* in creating your model?
    b. What did you learn as a group about the concepts of proportionality?
    c. What was the hardest part about this project?

Order form

You have a total of $15. We suggest you budget less than that so you have money left over in case you run out of building materials!

Item	**Price/pack**	**Number of packs**	**Price**
Popsicle sticks	$0.50 / pack of 25		
Straws	$0.75 / pack of 50		
Toothpicks	$0.50 / pack of 50		
		TOTAL (≤ $15)	

Please show calculations below to show how you determined how many packs of each item you need: