Lesson plan outline: Expected Value

by Stephen McCarthy

Before class:

1. Students watch video on Expected Value at Khan Academy (http://www.khanacademy.org/math/probability/v/expected-value--e-x) in preparation for class.

In class

2. Welcome students to class; do 2 minutes on "Math in the News": Justin Bieber's new single "Boyfriend" was just released – let's do some back-of-the-envelope calculations to see how many people we expect to buy it.

3. Take any burning questions students have about expected value.

4. Split into three groups by counting. Groups work at one of three activity stations, then rotate every 10 minutes or so (played by ear). Activity sheets are provided below.

    a. Gizmo Challenge: coins or darts? With ExploreLearning gizmos "Binomial Probabilities" and "Geometric Probability – Activity B", students use expected value to decide which of two games to play.

    b. Game Design Challenge: students design a casino game that will make money for the casino but also be exciting for players.

    c. Help & Homework: Teacher quizzes students to ensure they understand expected value fully, then they can get a start on the Expected Value homework.

5. Once each group has been at all three stations, gather class and debrief the two challenge activities:

    a. Gizmo challenge: ask for each group's result (Answer key: E(Coins) = 1.7 > E(Darts) = -0.7, so they should choose the coin game) and have one group briefly explain how they got the result.

    b. Game design challenge: have each group explain their game *without* revealing the expected value of playing. Put on board in chart, then have the class vote on which one they would like to play (excluding their own). After the vote, have groups reveal their expected values, and "hire" one group (balance most votes and best value).

Game Name	Class Votes	Expected value on $1 bid
Blackjack	5	-$0.067

After class

6. Homework: finish the expected value worksheet handed out in class; watch next Khan Academy video (Expected Value of Binomial Distribution).

Gizmo Challenge: Coins or Darts?

A friend has offered you a challenge to play one of two games, one with coins and one with darts. You know something about expected value and decide to use it to your advantage!

You must decide which game to play using the two gizmos open on the computers. You can use a third computer to do calculations (on a spreadsheet, for example).

Game 1: Darts

In the darts game, there is a 10" x 10" square dartboard, which has a circular target with radius 4" and a circular bull's-eye with radius 1". You will throw just one dart.

Here are the payoffs your friend offers:

- If you miss the board completely, you pay her $50.
- If you hit the board but miss the circle, you pay her $10.
- If you hit inside the circle but don't hit the bull's-eye, she pays you $10.
- If you hit the bull's-eye, she pays you $50.
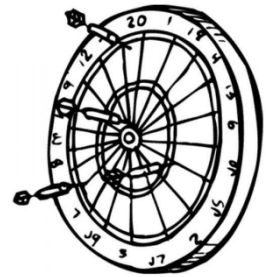
You know that you are a terrible dart thrower, so you figure you have a 5% chance of missing the board completely, and if you do hit the board your dart will land randomly on it.

Game 2: Coins

You will flip a coin four times. Here are the payoffs your friend offers:

- If all four coins are tails, you pay her $50.
- If three are tails and one is heads, you pay her $10.
- If there are two of each, you pay her $5.
- If three are heads and one is tails, she pays you $10.
- If all four are heads, she pays you $50.

You are actually pretty good at flipping coins, and have practiced a method which allows you to get heads 55% of the time.

Which game should you play?

Be prepared to explain how you got your answer to Mr. McCarthy or to the class.

Game Design Challenge

I am considering hiring your group to design a game for my casino.

There are only three requirements for the game:

- It must be legal and safe to do within a casino setting.
- It must be fun, i.e. people must want to play it.
- It must have an expected value that is profitable for the casino.

Use the rest of this paper to describe your game and demonstrate that it will be profitable for my casino – please hand it in to Mr. McCarthy when done. I will hire a group based on the three criteria above.

Game description (use words or drawings as necessary):

Proof that game is profitable (please show mathematically):

Expected payoff for casino on a $1 player bid: $ ___._____

Expected Value worksheet

Please complete all work in your workbooks.

1. On a multiple-choice test, a student is given five possible answers for each question. The student receives 1 point for a correct answer and loses ¼ point for an incorrect answer. If the student has no idea of the correct answer for a particular question and merely guesses, what is the student's expected gain or loss on the question?

   Suppose also that on one of the questions you can eliminate two of the five answers as being wrong. If you guess at one of the remaining three answers, what is your expected gain or loss on the question?

2. A dairy farmer estimates for the next year the farm's cows will produce about 25,000 gallons of milk. Because of variation in the market price of milk and cost of feeding the cows, the profit per gallon may vary with the probabilities given in the table below. Estimate the profit on the 25,000 gallons.

Gain per gallon	$1.10	$0.90	$0.70	$0.40	$0.00	-$0.10
Probability	30%	38%	20%	6%	4%	2%

3. At many airports, a person can pay only $1.00 for a $100,000 life insurance policy covering the duration of the flight. In other words, the insurance company pays $100,000 if the insured person dies from a possible flight crash; otherwise the company gains $1.00 (before expenses). Suppose that past records indicate 0.45 deaths per million passengers. How much can the company expect to gain on one policy? On 100,000 policies?

4. A construction company wants to submit a bid for remodeling a school. The research and planning needed to make the bid cost $4000. If the bid were accepted, the company would make $26,000. Would you advise the company to spend the $4000 if the bid has only 20% probability of being accepted? Explain your reasoning.

5. Suppose the warranty period on your family's new television is about to expire and you are debating about whether to buy a one-year maintenance contract for $35. If you buy the contract, all repairs for one year are free. Consumer information shows that 12% of the televisions like yours require an annual repair that costs $140 on the average. Would you advise buying the maintenance contract? Explain your reasoning.